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Review of AISI Design Guide for Cold-Formed Steel Purlin Roof Framing Systems – Component Stiffness Method

Michael W. Seek, PE¹

Abstract

This paper reviews the Component Stiffness Method for determining anchorage forces in roof systems as presented in Chapter 5 of the new AISI *Design Guide for Cold-Formed Steel Purlin Roof Framing Systems*. The four steps in the general methodology of the component stiffness method are presented. First, the development of forces requiring anchorage in purlin roof systems is discussed. Next, calculation of the stiffness of each of the components of the roof system and the procedure for distributing these forces through the system is presented. The final step, evaluation of the effectiveness of the anchors in preventing deformation of the system, is discussed. The five numerical examples relating to the component stiffness method are outlined.

Introduction

In June 2009, the American Iron and Steel Institute published the *Design Guide for Cold-Formed Steel Purlin Roof Framing Systems* (2009). This publication originated primarily in support of the new roof bracing and anchorage provisions in the AISI *North American Specification for the Design of Cold-Formed Steel Structural Members* (2007). These provisions, now in Section D6.3.1, represent a new approach to quantifying anchorage forces using a systematic stiffness analysis of the roof system. The *Specification* outlines one method for performing the stiffness analysis. However, recognizing that there are a number of different ways to perform such an analysis, the *Specification* allows the use of other methods to determine the anchorage forces. The AISI *Design Guide* contains details for several methods to determine anchorage forces: Simplified Specification Solution, Matrix Solution, Frame Element Stiffness Model and Shell Element Stiffness Model and the Component Stiffness Method. As each method increases in computational requirements, each allows for more refined analyses on more complex systems. Table 1.1 in the *Design Guide* provides a matrix of applicability for each method.

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The *AISI Design Guide* is arranged in five chapters. The first chapter provides an introduction to the components comprising typical cold-formed steel roof systems. The second chapter provides a discussion of purlin design using the R-factor method for through fastened systems subjected to uplift loading and the Base Test Method for standing seam systems. In Chapter 3, design assumptions for continuous span purlin design are discussed and two design examples are provided. Chapter 4 presents the new purlin anchorage provisions in the Specification with several design examples. In addition, a Simplified Specification Solution and a Matrix Solution are also introduced. Alternate analysis procedures are presented in Chapter 5. The bulk of this chapter is devoted to explaining the mechanics of purlin anchorage, the development of the Component Stiffness Method, and several examples. Chapter 5 also presents guidelines for determining anchorage forces using a frame element finite element model and a shell element finite element model.

The Component Stiffness Method can be applied to solve anchorage forces for single or multiple span systems with supports, third point, midpoint, supports plus third point lateral restraints and supports plus third point torsional restraints. The method is versatile and provides a thorough representation of the system of purlins. However, to provide this versatility and account for the variety of systems provided by the different manufacturers, the method requires the designer to apply more properties of the purlin, the sheathing and the connections between the purlin and sheathing and purlin and rafters. The increased complexity allows for a refined analysis.

The component stiffness method is fundamentally a stiffness analysis. To perform the analysis, there are 4 steps. The first is to determine the external forces acting at each node on the system. The second is to determine the stiffness of the system. Once the nodal forces and stiffness of the system is determined, forces can be distributed throughout the system according to stiffness. One important final step is to perform serviceability checks to evaluate the effectiveness of the anchors.

Forces in the System

In purlin supported roof systems, the load carrying capacity of a purlin is affected by its attachment to the sheathing. Purlins are designed based on the assumption of constrained bending. That is, despite the fact that Z-sections have rotated principal axes relative to their normal orthogonal axes and sloped roof systems are subjected to torsional downslope loading, the sheathing is assumed to restrict the lateral and rotational movements of the purlin, constraining

bending of the purlin to a plane perpendicular to the sheathing. Through-fastened systems are assumed to perfectly restrain the purlin when subjected to gravity loading. Standing seam systems have greater flexibility and are typically not as effective as a through fastened system at constraining the bending. Consequently, design of purlins attached to standing seam systems is based upon the Base Test Method. With the Base Test Method, a reduction is applied to the fully constrained bending strength in the form of an R-Factor. For the sheathing to constrain the bending of a purlin, forces are developed in the sheathing. Therefore, to insure validity of the assumption of constrained bending, the forces developed in the sheathing must be anchored externally. The Component Stiffness Method is a method of determining this anchorage force that closely mimics this interaction between the purlin and sheathing.

Each purlin, by virtue of the restraint provided by the sheathing, generates a force that must be resisted by the anchorage device. As gravity loads are applied, the sheathing attached to the top flange of the purlin partially restrains lateral and torsional movements of the purlin. Forces generated as a result of the interaction between the purlin and the sheathing must be transferred through the sheathing to the anchorage device. The first step in the Component Stiffness Method is to determine this force.

The interaction between a purlin and sheathing is complex. Resistance to lateral movement is a function of the diaphragm stiffness of the sheathing, G' , which includes float in standing seam clips. Torsional resistance provided by the sheathing is affected by the type of fastener (standing seam clip or through fastened), by the location of the fasteners between sheathing and purlin, gage of purlin and sheathing material, and the presence of insulation. In the Component Stiffness Method, the connection between the purlin and sheathing is represented by a spring. The stiffness of the spring, k_{mclip} , is defined as the moment generated in the connection between the purlin and sheathing per unit torsional rotation of the purlin per unit length along the purlin.

To determine the force contributed by each purlin, displacement compatibility of the top flange of the purlin at midspan is considered. The greater the lateral and torsional restraint provided by the sheathing, the greater the anchorage force. As the restraint of the sheathing is reduced, the less anchorage force is generated as the purlin deviates from constrained bending. Consider the following example of a simple span purlin subjected to a uniformly applied gravity loading. In absence of the restraining effects of the sheathing, the Z-section, because of its rotated principal axes, when loaded uniformly in the plane of its web deflects laterally as shown in Figure 1 (a). In typical roof systems, the uniformly applied load is assumed to act at an eccentricity at the top flange (δb) causing an upslope

rotation of the purlin. The total lateral and torsional displacement of the purlin unrestrained by the sheathing is shown in Figure 1 (a).

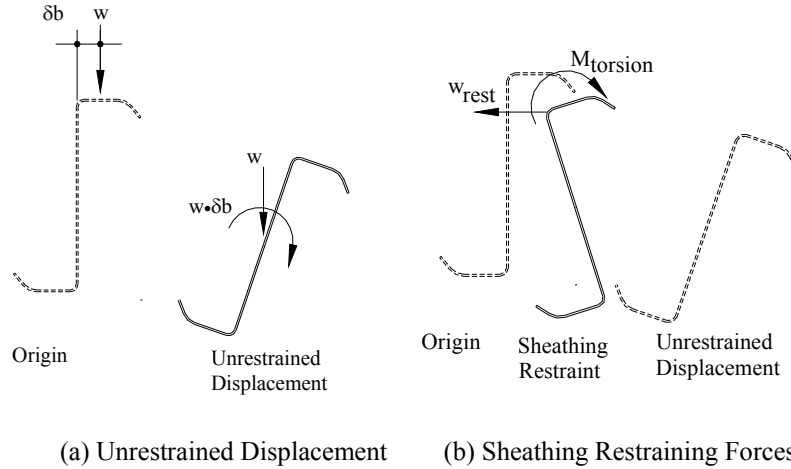


Figure 1 Restraining Effect of Sheathing on Purlin Displacement

Figure 1 (b) shows the lateral and torsional restraining effects of the sheathing. As the purlin moves laterally, uniform resistance is provided by the diaphragm action in the sheathing. This effect is represented by a uniform horizontal load in the plane of the sheathing, w_{rest} . This horizontal load has the effect of pushing the purlin downslope towards its original undeflected position. Because this horizontal load is applied at the top flange of the purlin, it causes a downslope rotation of the purlin. The sheathing also resists torsional rotations of the purlin through the development of a moment, $M_{torsion}$, in the connection between the purlin and the sheathing.

By equating the deformation of the purlin in the absence of the sheathing with the restoring displacement provided by the sheathing, the uniform restraint force in the sheathing, w_{rest} , is determined. For a single span purlin with supports restraints,

$$w_{rest} = w \cdot \sigma \quad (1)$$

Where

$$\sigma = \frac{5 \left(\frac{I_{xy}}{I_x} \cos \theta \right) L^4}{384 EI_{my}} + \frac{((\delta b + m) \cos \theta) d}{2} \tau + \frac{L^2 \sin \theta}{8 G' \text{ Bay}}}{\frac{5 L^4}{384 EI_{my}} + \frac{d^2}{4} \tau + \frac{L^2}{8 G' \text{ Bay}}} \quad (2)$$

If the purlin is rigidly restrained by the sheathing, that is the sheathing prevents horizontal movement and torsional rotation, then $\sigma = I_{xy}/I_x$, and the purlin conforms to constrained bending. Typically, σ will range between I_{xy}/I_x (perfectly restrained) for a very rigid diaphragm and purlin-sheathing connection to zero, where no restraint is provided by sheathing. There are a few instances, such as high slope roofs or downslope facing purlins where $\sigma > I_{xy}/I_x$. Note that the uniform restraint force that is generated in the sheathing is resolved in the sheathing. The uniform restraint force along the length of the purlin is counteracted by a force at frame lines equal to $w_{\text{rest}} \cdot L/2$ as shown in Figure 2. The uniform restraint force in the sheathing is not directly transferred into the anchorage force.

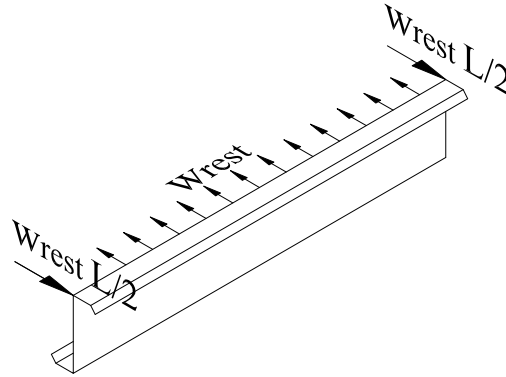


Figure 2 Uniform restraint force in sheathing

To determine the force each purlin adds to the system, moments are summed about the base of the purlin based on the free body diagram shown in Figure 3. For a sloped roof, the components of the gravity load are divided into a normal component, $w \cos \theta$, perpendicular to the plane of the sheathing and a downslope component, $w \sin \theta$, in the plane of the sheathing. The torsional moment,

M_{torsion} , is the moment that is generated in the connection between the purlin and the sheathing as the sheathing resists the tendency of the purlin to twist. For a supports restraint configuration, the torsional moment is calculated based on the torsional rotation of the purlin at midspan.

$$M_{\text{torsion}} = \frac{2}{3} k_{\text{mclip}} \cdot wL \left(\sigma \frac{d}{2} - (\delta b) \cos \theta \right) \tau$$

Where

$$\tau = \frac{\frac{a^2 \beta}{GJ}}{1 + k_{\text{mclip}} \frac{\kappa}{GJ}}$$

Both laboratory testing and finite element models have shown bending of the top flange and subsequent deformation of the purlin cross section. For thinner purlins, as the purlin twists, less of the torsion is transferred to a moment in the connection between purlin and sheathing. To account for this local deformation and its effect on the anchorage force, a moment, M_{local} , is incorporated into the Component Stiffness Method.

$$M_{\text{local}} = -wL \cdot \delta b \cos \theta \frac{k_{\text{mclip}}}{k_{\text{mclip}} + \frac{Et^3}{3d}}$$

Summing moments about the base of the free body diagram shown in Figure 3, the net overturning effects are distilled into P_i , where

$$P_i = \frac{wL}{d} (\delta b \cos \theta - d \sin \theta) + M_{\text{torsion}} + M_{\text{local}}$$

The torsional moment varies for each restraint configuration and for single and multi-span configurations. In Section 5.1.6 of the *AISI Design Guide*, a summary of the equations required for the different restraint and span configurations is provided.

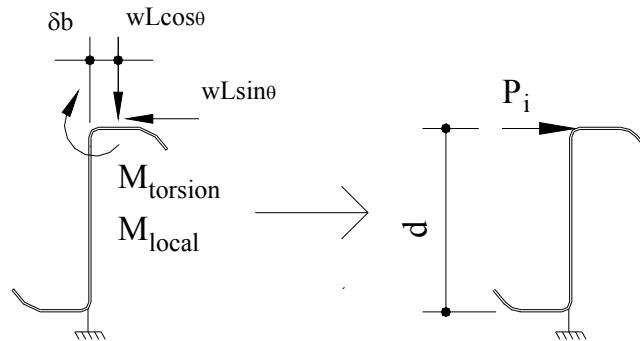


Figure 3 Free Body Diagram of Purlin Overturning Forces

Stiffness of Components

Resistance to the overturning forces generated by each purlin is provided mostly by the externally applied anchors and to a lesser extent by the connection between the purlin and sheathing and the connection between the purlin and rafter. The purlin overturning forces are distributed to each of these “components” of the system according to the relative stiffness of each. It is necessary, therefore, to quantify the stiffness of each of the components.

By determining the stiffness of each of the components in the system, the designer has greater flexibility and the result is a better approximation of the roof system. The component stiffness method allows the designer to account for the different stiffness of various purlin support conditions, effects of clip type and insulation.

Most of the total stiffness of a system of purlins comes from the anchorage devices. In the component stiffness method, anchorage devices are divided into two categories: support and interior. The stiffness of the anchorage is defined as the force developed in the anchor relative to the lateral displacement of the top flange at the anchorage device. Support anchors are subdivided into either an antiroll anchorage device or a discrete anchor. A discrete anchor is considered to only restrain the web of the purlin at a single point along the height of the web whereas an antiroll anchorage clamps the web at multiple locations along its height. Both types of anchorage are shown in Figure 4.

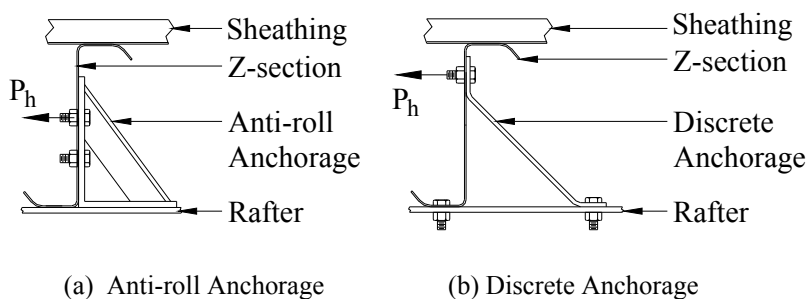


Figure 4 Types of Support Anchorage Devices

Typically, there is flexibility in the web of the purlin between the top of the support anchor and the top flange of the purlin (see Figure 5). Therefore, for a supports anchorage configuration, the stiffness is the combined stiffness of the anchorage device and stiffness of the web of the purlin between the top of the anchorage device and top flange of the purlin. For interior restraints, flexibility of the purlin web is not considered and the stiffness of an interior restraint is simply the stiffness of the anchorage device itself. The *AISI Design Guide* provides derivations and equations for several anchor configurations.

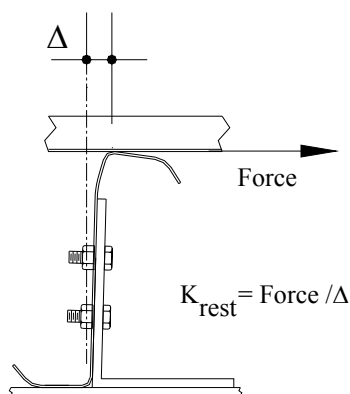


Figure 5 Stiffness of Anchorage Device

By virtue of its connection to the sheathing and connection to the rafter, a purlin has some inherent resistance to overturning. This inherent resistance is known

as the system effect. As a purlin is subjected to overturning, a moment is developed in the connection between the purlin and sheathing. The moment is proportional to the lateral deflection of the top flange. Therefore, the component of the sheathing stiffness, K_{shg} is defined as the moment developed in the connection between the purlin and sheathing along the entire span of the purlin per unit lateral displacement of the top flange at the restraint location (see Figure 6) The sheathing stiffness is a function of type of connection between the purlin and sheathing, purlin span, thickness, and torsional properties. Procedures for determining sheathing stiffness are provided in the *AISI Design Guide*.

Similarly, for the connection of the purlin to the rafter, whether it is a flange bolted connection or a web plate, as overturning of the purlin occurs, a moment, M_{rafter} , is developed. The stiffness of the rafter connection is defined as the moment generated at the rafter per unit lateral displacement of the top flange of the purlin. The *AISI Design Guide* provides equations to approximate the stiffness for both flange bolted and web plate connections.

With the component stiffness method, anchorage forces are analyzed per line of restraint. The line of restraint includes all purlins in the bay. For example, a three-span continuous purlin system with anchors at the frame lines has 4 lines of restraint: one at each of the exterior frame lines and one for each interior frame line. For solution of the anchorage forces, the entire stiffness along the line of restraint is considered. This stiffness includes the stiffness of the anchors, stiffness of the purlin-sheathing connection tributary to the line of restraint, and the stiffness of the rafter connections. The stiffness included from the rafter connection includes all locations that do not have a support anchor. For interior restraint configurations (midpoints and third points) the rafter stiffness is conservatively ignored.

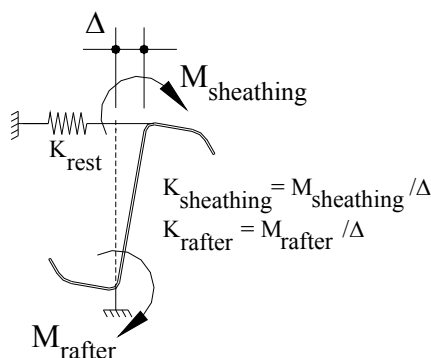


Figure 6 Stiffness of Rafter and Sheathing Components

Anchorage Force Determination

To solve for anchorage forces using the Component Stiffness Method, at each line of anchorage, the system of purlins is considered to have a single degree of freedom: the lateral displacement of the top flange at the restraint location. The sheathing or some other mechanism such as in strapping is assumed to rigidly link the purlins at the line of anchorage, so each purlin along the line of anchorage has the same lateral deflection. Because the stiffness of each component (anchors, purlin-sheathing connection and purlin-rafter connection) is related to this lateral deflection, forces are distributed throughout the system according to the relative stiffness. The total overturning force acting at the line of anchorage is the sum of the forces each purlin contributes to the system, P_i . The sum of these forces is then distributed to each anchor according to the stiffness of the anchor, K_{rest} , relative to the total stiffness at the line of anchorage, K_{total} (see Figure 7). In general, the anchorage force, P_L , is

$$P_L = \sum_{n_p} P_i \cdot \frac{K_{rest}}{K_{total}}$$

Anchorage force is a function of the height of the application of restraint. The stiffness of the anchor is affected by the height of restraint. Typically the lower the restraint from the top flange, the less the stiffness, which will typically reduce the anchorage force. However, since the anchorage force is determined by summing moments about the base of the purlin, as this moment arm is reduced, the anchorage force will increase by a factor of d/h . The anchorage force calculated at the height of the restraint, P_h , is

$$P_h = P_L \frac{d}{h}$$

In terms of anchorage force, the decrease in stiffness and decrease in moment arm will often negate each other. However, as the location of the anchorage is lowered from the top flange, there is an increase in the lateral movement of the top flange. Because the purpose of providing anchorage is to limit lateral deflection, it is recommended that anchorage be provided as close as possible to the top flange.

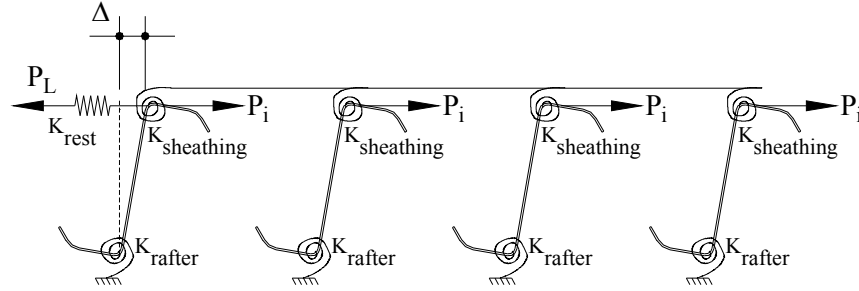


Figure 7 Anchor Force Along Line of Anchorage

Anchorage Effectiveness

Anchorage force is a function of the stiffness of an anchor relative to the stiffness of the system. As anchor stiffness is reduced relative to the system stiffness, the anchorage force is reduced. However, as anchor stiffness is reduced, lateral deflection of the purlin top flange increases. Therefore, to prevent excessive flexibility in a system of purlins, deflection limits were established in the 2007 *AISI Specification*. The *Specification* sets the following limits on the lateral movement of the top flange of a purlin at the line of anchorage.

$$\Delta_{tf} \leq \frac{1}{\Omega} \frac{d}{20} \quad (\text{ASD})$$

$$\Delta_{tf} \leq \phi \frac{d}{20} \quad (\text{LRFD, LSD})$$

The lateral deflection of the top flange of a purlin along the line of anchorage using the component stiffness method is

$$\Delta_{tf} = \frac{P_L}{K_{rest}}$$

Because the system of purlins depends upon the sheathing to partially restrain movements and transfer loads to the anchors, the sheathing must have sufficient diaphragm stiffness. For most bracing situations, the *Specification* limits the lateral deflection of the top flange of the purlin between lines of anchorage to

L/360. For bracing configurations where third point torsional braces are used in conjunction with lateral restraints along the frame lines, the lateral deflection limits are relaxed to L/180.

Away from the lines of anchorage, lateral deflection is a function of the uniform restraint force in the sheathing, $w \cdot \cos(\theta) \cdot \sigma$, and the downslope component of the applied load, $w \cdot \sin \theta$. For supports and supports plus third point torsional braces, maximum lateral deflection occurs at midspan of the purlin between frame lines. For low slope roofs, deflection will typically be upslope (considered a positive deflection) and as the slope of the roof increases, lateral deflection will shift downslope (negative deflection).

For interior restraint configurations (midpoints and third points), lateral deflection between lines of anchorage is checked at the frame lines. Lateral deflection will typically be negative (downslope) for low slope roofs with interior restraints. As the roof slope increases, the downslope lateral deflections will increase. Equations to calculate the lateral displacement between lines of anchorage are provided in Section 5.1.6 of the *AISI Design Guide*.

For supports plus third point lateral anchors, the lateral displacements between anchors will typically be very small, so lateral deflection is checked at the third point anchors using the above equation for top flange deflection at a restraint. The lateral deflection at the third point is compared to L/360.

The last aspect of anchor effectiveness that needs to be checked is the transfer of force from the sheathing to purlin, P_{sc} . At anchorage locations, the connection between the sheathing and the purlin must transfer significant loads. In many cases (particularly at anchors along the frame lines) this force will exceed the anchorage force. Although the *Specification* does directly address the connection, considering the magnitude of the forces transferred, the designer should acknowledge the need for a mechanism to transfer the force from the sheathing to purlin. The force P_{sc} varies for each restraint configuration, so equations for each configuration are provided in Section 5.1.6 in the *AISI Design Guide*.

Examples

The Component Stiffness Method is a complex solution to a complex problem that accounts for the many variables that affect anchorage forces. As such, the calculations are facilitated with the help of a computer. In the *AISI Design Guide*, five examples are provided. The first four examples (Examples 8 -11) are based upon a four-span continuous system of Z-sections with standing seam

sheathing. In each of the different examples, the system of purlins is subjected to different bracing configurations. Example 8 demonstrates the system of purlins with antiroll anchorage devices along the frame lines. In Example 9 anchorage is provided by third point anchors applied at the eave of the system of purlins. Example 10 demonstrates the calculation of anchorage forces for supports plus third point torsional braces. Example 11 takes the same third point anchors used in Example 9 but shows the effects of providing additional restraint along the frame lines. The last example has the same general roof configuration as Examples 8-11 but C-sections are used instead of Z-sections. The C-sections are lapped over the interior frame lines and anchorage is provided in the form of antiroll anchorage devices intermittently along the frame line.

Conclusion

The new *AISI Design Guide for Cold-Formed Steel Purlin Roof Framing Systems* is an indispensable source for the designer of purlin roof systems. The *Design Guide* provides insight into design methods and assumptions for purlins but the bulk of the guide is devoted to calculation of purlin anchorage forces, both the adopted by the *Specification* as well as several alternate methods allowed by the *Specification*. Among the alternate methods, the Component Stiffness Method is derived and described in detail. With a summary of equations and examples applying the Component Stiffness Method, the *Design Guide* provides valuable insight and analysis for purlin supported roof systems.

Appendix - References

AISI (2007) *North American Specification for the Design of Cold-Formed Steel Structural Members, 2007 Edition*, American Iron and Steel Institute, Washington, D.C.

AISI (2009) *Design Guide for Cold-Formed Steel Purlin Roof Framing Systems*, American Iron and Steel Institute, Washington, D.C.

Appendix – Notation

a	Torsional constant $\sqrt{\frac{EC_w}{GJ}}$
b	Width of C- or Z-section top flange (in.) (mm)
Bay	Total width of diaphragm perpendicular to span (ft) (m)
C_w	Torsional warping constant of cross-section (in. ⁶) (mm ⁶)
d	Depth of C- or Z-section (in.) (mm)
E	Modulus of elasticity (29,500,000 psi) (203,400 MPa)
G	Shear modulus (11,300,00 psi) (78,000 MPa)
G'	Diaphragm shear stiffness. Ratio of shear per foot to the deflection per unit width of diaphragm assembly. (lb/in.) (N/m)
h	Height of applied restraint measured from base of purlin parallel to web (in.) (mm)
I_{my}	Modified moment of inertia $\frac{I_x I_y - I_{xy}^2}{I_x}$
I_x	Moment of inertia of full unreduced section about axis perpendicular to the plane of the web (in. ⁴) (mm ⁴)
I_{xy}	Product of inertia of full unreduced section about major and minor centroidal axes (in. ⁴) (mm ⁴)
J	Saint-Venant torsion constant (in. ⁴) (mm ⁴)
k_{mclip}	Combined rotational stiffness of sheathing and connection between the purlin and sheathing per unit length along span of purlin (lb-in./ft) (N-m/m)
K_{rafter}	Moment developed in connection between purlin and rafter per unit lateral displacement of top flange of purlin at restraint (lb-in./in.) (N-m/m)
K_{rest}	Force restrained at top flange of purlin per unit lateral displacement of top flange at restraint location (lb/in.) (N/m)
K_{shtg}	Moment developed in connection between purlin and sheathing per

	unit lateral displacement of top flange of purlin at restraint (lb-in./in.) (N-m/m)
K_{total}	Total stiffness of system at anchor location.
L	Span of purlin (ft) (m)
m	Horizontal distance from shear center of C-section to mid-plane of web ($m = 0$ for Z-sections) (in.) (mm)
M_{local}	Moment developed in sheathing due to cross sectional deformation of purlin (lb-in.) (N-m)
M_{rafter}	Moment developed in connection between rafter and purlin due to lateral movement of top flange relative to base (lb-in.) (N-m)
M_{shtg}	Moment developed in sheathing along the span of the purlin due to lateral movement of top flange relative to base (lb-in.) (N-m)
$M_{torsion}$	Moment developed in sheathing due to twist of purlin relative to sheathing (lb-in.) (N-m)
P_h	Anchorage force per anchorage device at height of restraint (lb) (N)
P_i	Overturning force generated per purlin per half span (lb) (N)
P_L	Anchorage force per anchorage device at top of purlin (lb) (N)
P_{sc}	Shear force in connection between purlin and sheathing at anchorage location (lb) (N)
n_p	Number of purlins in a bay
w	Uniform loading on purlin (lb/ft) (N/m)
w_{rest}	Uniform diaphragm restraint force provided by sheathing (lb/ft) (N/m)
α	Coefficient for purlin direction
β	Torsional constant for beam subjected to uniform torsion (rad.)
δ	Coefficient for determining load eccentricity on purlin top flange (1/3)
Δ_{tf}	Horizontal deflection of the top flange of purlin at restraint (in.) (mm)
η	Number of up slope facing purlins minus the number of down slope facing purlins
κ	Torsional constant for beam subjected to parabolically varying torsion (rad·in. ²) (rad·mm ²)
σ	Proportion of uniformly applied load transferred to a uniform restraint force in the sheathing
θ	Angle between the vertical and the plane of the purlin web (degrees)
τ	Torsional constant for beam subjected to uniform torsion with uniformly distributed rotational springs resistance (rad/lb) (rad/N)

